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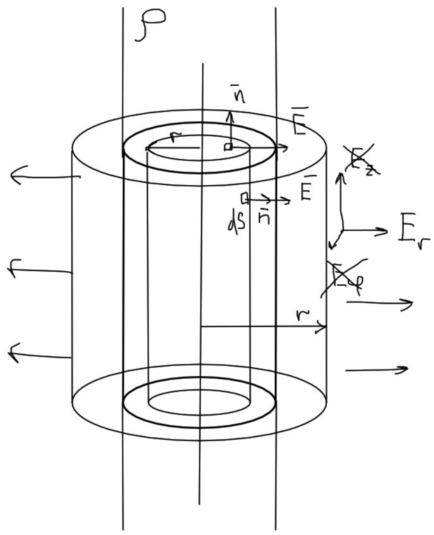
$$\oint \vec{E}\vec{n}dS = \frac{Q}{\varepsilon_0} = \frac{\int \rho dV + \int \sigma dS + \sum q_i}{\varepsilon_0} \tag{1.1}$$



$\oint \bar{E}\,\bar{n}\,dS$

У внутрішній області (r<R)

$$\oint \vec{E}d\vec{S} = \int_{S_б} EdS = \iint E(r) r d\varphi dz = E(r)\int_{S_б} dS = E\cdot S_б = E\cdot 2\pi r H =$$
$$= \frac{\iiint \rho\cdot dz\cdot rd\varphi\cdot dr}{\varepsilon_0} = \frac{\rho H 2\pi}{\varepsilon_0}\int_0^r rdr = \rho\frac{\pi r^2 H}{\varepsilon_0} \tag{1.2}$$

$$E_{in} = \frac{\rho r}{2\varepsilon_0} \tag{1.3}$$

У зовнішній області (r>R)

$$\oint EdS = \int_{S_б} EdS = \iint E(r) r d\varphi dz = E(r)\int_{S_б} dS = E\cdot S_б = E\cdot 2\pi r H =$$
$$= \frac{\iiint \rho\cdot dz\cdot rd\varphi\cdot dr}{\varepsilon_0} = \frac{\rho H 2\pi}{\varepsilon_0}\int_0^R rdr = \rho\frac{\pi R^2 H}{\varepsilon_0} \tag{1.4}$$

$$E_{ex} = \frac{\rho R^2}{2\varepsilon_0 r} \tag{1.5}$$

Потенціал

$$\Delta\varphi = \frac{\Delta U}{q} = \frac{-A}{q} = \frac{-\int \vec{F}d\vec{r}}{q} = -\int \vec{E}d\vec{r}$$

$$\varphi(\vec{r}_2) - \varphi(\vec{r}_1) = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E}d\vec{r} \tag{1.6}$$

$$\varphi(\vec{r}) = -\int \vec{E}d\vec{r} + C$$

(r<R)

$$\varphi_{in} = -\int E_{in}dr + C_1 = -\int \frac{\rho r}{2\varepsilon_0}dr + C_1 = -\frac{\rho r^2}{4\varepsilon_0} + C_1 \tag{1.7}$$

(r>R)

$$\varphi_{ex} = -\int \frac{\rho R^2}{2\varepsilon_0 r} dr + C_2 = -\frac{\rho R^2}{2\varepsilon_0} \ln r + C_2 \quad (1.8)$$

Умова неперервності

$$\varphi_{in}(R) = \varphi_{ex}(R) = 0$$

$$-\frac{\rho R^2}{4\varepsilon_0} + C_1 = -\frac{\rho R^2}{2\varepsilon_0} \ln R + C_2 = 0 \quad (1.9)$$

$$C_1 = \frac{\rho R^2}{4\varepsilon_0}$$

$$C_2 = \frac{\rho R^2}{2\varepsilon_0} \ln R \quad (1.10)$$

$$\varphi_{in}(r) = \frac{\rho}{4\varepsilon_0}(R^2 - r^2)$$

$$\varphi_{ex}(r) = \frac{\rho}{2\varepsilon_0} \ln \frac{R}{r} \quad (1.11)$$

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$$\vec{E} = -grad\varphi = -\left(\frac{\partial \varphi}{\partial x} \quad \frac{\partial \varphi}{\partial y} \quad \frac{\partial \varphi}{\partial z} \right) \quad (1.12)$$

$$E_x = -\frac{\partial}{\partial x}\left(a_x x + a_y y + a_z z \right) = -a_x$$

$$E_y = -\frac{\partial \varphi}{\partial y} = -a_y \quad (1.13)$$

$$\vec{E} = -\vec{a} \quad (1.14)$$

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b)

$$\vec{E} = -grad\varphi \quad (1.15)$$

$$E_x = 2axy = -\frac{\partial \varphi}{\partial x}$$

$$E_y = a(x^2 - y^2) = -\frac{\partial \varphi}{\partial y} \quad (1.16)$$

$$\int 2axydx = \int -d\varphi$$

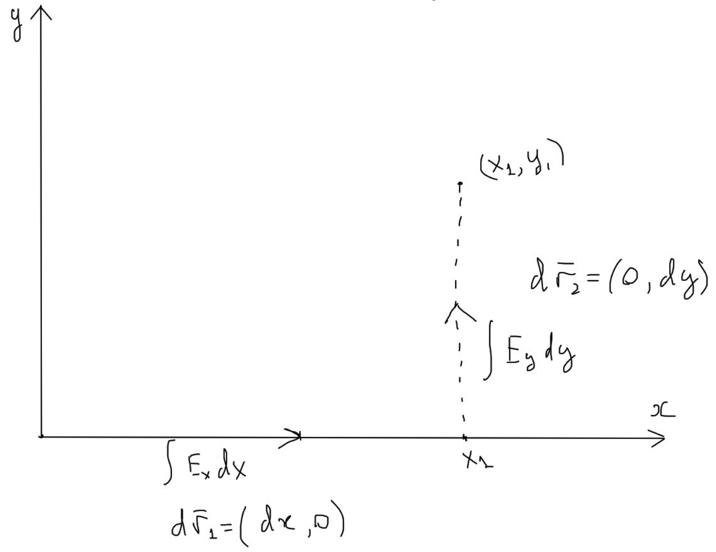
$$ax^2 y + C_1(y) = -\varphi \quad (1.17)$$

$$\int a(x^2 - y^2)dy = \int -d\varphi$$

$$ax^2 y - a\frac{y^3}{3} + C_2(x) = -\varphi \quad (1.18)$$

$$\varphi = -ax^2 y + a\frac{y^3}{3} + C_2 \quad (1.19)$$

$$\Delta\varphi = -\int \bar{E}\, d\bar{r} = -\int E_x dx + E_y dy$$



$$\varphi(x_1,y_1)-\varphi(0,0) = -\int_0^{x_1} E_x(x,0)dx - \int_0^{y_1} E_y(x_1,y)dy = \\ = -\int_0^{x_1} 2ax\cdot 0\cdot dx - \int_0^{y_1} a(x_1^2-y^2)dy = -ax_1^2y_1 + a\frac{y_1^3}{3} \tag{1.20}$$

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$$\vec{E} = -grad\psi = -\left(\frac{\partial\psi}{\partial r};\frac{1}{r}\frac{\partial\psi}{\partial\theta};\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\varphi}\right) \tag{1.21}$$

$$\vec{E} = -\left(\frac{\partial}{\partial r}\frac{e^{-\lambda r}}{r};0;0\right) = -\left(\frac{-\lambda e^{-\lambda r}r - e^{-\lambda r}}{r^2};0;0\right) = e^{-\lambda r}\left(\frac{1+\lambda r}{r^2};0;0\right) \tag{1.22}$$

$$\int EdS = E\int dS = E4\pi r^2 = \frac{1}{\varepsilon_0}\int \rho dV = \frac{1}{\varepsilon_0}\int \rho 4\pi r^2 dr \tag{1.23}$$

$$\frac{d}{dr}\left(e^{-\lambda r}\frac{1+\lambda r}{r^2}4\pi r^2\right) = \frac{1}{\varepsilon_0}\rho 4\pi r^2$$

$$\frac{d}{dr}\left(e^{-\lambda r}(1+\lambda r)\right) = e^{-\lambda r}(-\lambda)(1+\lambda r) + e^{-\lambda r}\lambda = -\lambda^2 r e^{-\lambda r} = \frac{1}{\varepsilon_0}\rho r^2 \tag{1.24}$$

$$\rho = \frac{-\lambda^2\varepsilon_0 e^{-\lambda r}}{r}$$

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