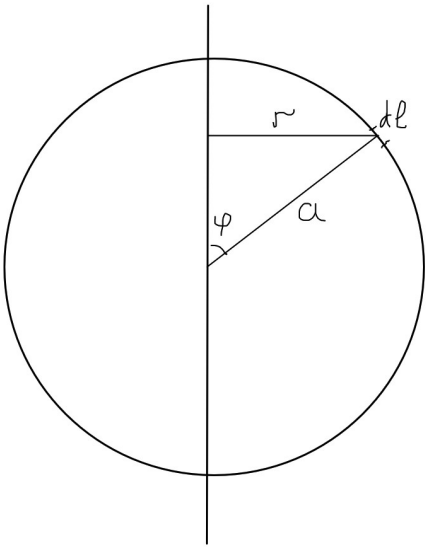
Динаміка руху твердого тіла

$$\vec{N} = \frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt}$$
$$\vec{F} = m\frac{d\vec{v}}{dt}$$
$$\vec{N} = [\vec{r} \times \vec{F}] \qquad (1.1)$$
$$\vec{L} = [\vec{r} \times \vec{p}] = I\vec{\omega}$$
$$I = mr^2$$

Іродов 1.281



Формула обрахунку моменту інерції

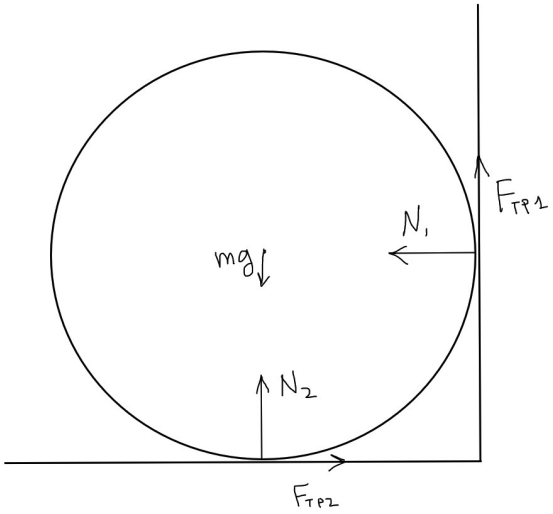
$$I = \int dm \cdot r^2 \qquad (1.2)$$
$$\frac{dm}{m} = \frac{dl}{2\pi a}$$
$$\frac{dm}{dl} = \frac{m}{2\pi a} = \rho$$

(1.3)

$$\frac{dm}{dS} = \frac{m}{a^2} = \rho$$
$$dm = \rho dV$$
$$dm = \rho dS$$

$$I = \int \frac{m}{2\pi a} dl \cdot (a\sin\varphi)^2 = \frac{ma}{2\pi}\int dl \cdot \sin^2\varphi = \frac{ma}{2\pi}\int_0^{2\pi} ad\varphi \cdot \sin^2\varphi =$$
$$= \frac{ma^2}{2\pi}\int_0^{2\pi} d\varphi \cdot \frac{1-\cos 2\varphi}{2} = \frac{ma^2}{2\pi}\left(\frac{\varphi}{2} - \frac{\sin 2\varphi}{4}\right)\Bigg|_0^{2\pi} = \frac{ma^2}{2} \qquad (1.4)$$

Іродов 1.295



2-й закон Ньютона

$$-N_1 + F_{mp2} = 0$$
$$F_{mp1} - mg + N_2 = 0 \quad (1.5)$$

$$F_{mp1} = kN_1$$
$$F_{mp2} = kN_2 \quad (1.6)$$

$$-N_1 + kN_2 = 0$$
$$kN_1 - mg + N_2 = 0 \quad (1.7)$$

$$k^2 N_2 + N_2 = mg \quad (1.8)$$

$$N_2 = \frac{mg}{1+k^2} \quad (1.9)$$

$$F_{mp1} = k\frac{kmg}{1+k^2} = \frac{k^2}{1+k^2}mg$$
$$F_{mp2} = k\frac{mg}{1+k^2} \quad (1.10)$$

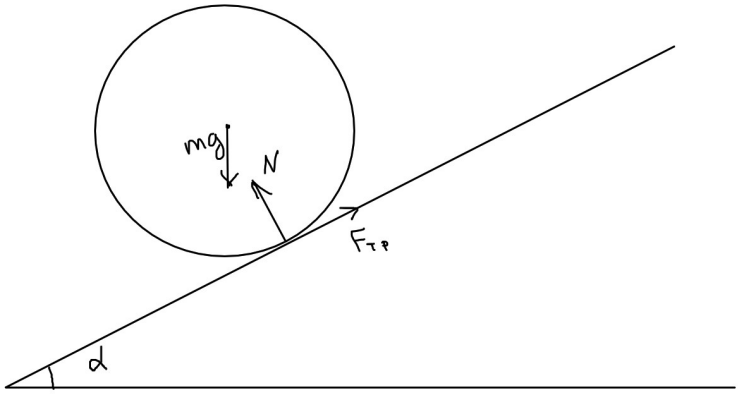
Рівняння моментів:

$$RF_{mp1} + RF_{mp2} = I\frac{d\omega}{dt} = \frac{mR^2}{2}\beta \quad (1.11)$$

$$\beta = \frac{2kg(1+k)}{R(1+k^2)} \quad (1.12)$$

$$t = \frac{\omega_0}{\beta} = \frac{\omega_0 R(1+k^2)}{2kg(1+k)}$$
$$\omega = \omega_0 - \beta t = 0 \quad (1.13)$$

КШФ 8.10



2-й закон Ньютона

$$\begin{aligned} -F_{mp} + mg\sin\alpha &= ma \\ -mg\cos\alpha + N &= 0 \end{aligned} \tag{1.14}$$

Рівняння моментів:

$$F_{mp}R = I\beta \tag{1.15}$$

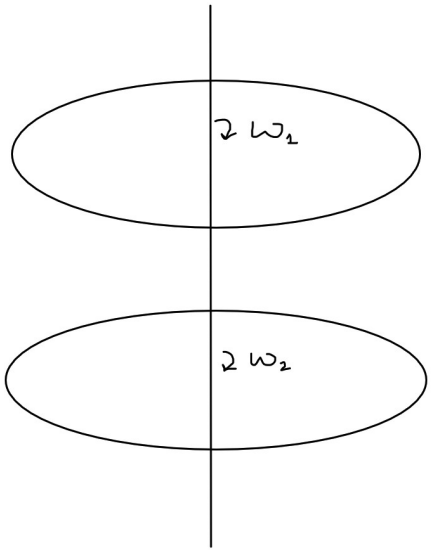
Умова не проковзування:

$$\beta R = a \tag{1.16}$$

$$F_{mp} = \frac{I\beta}{R} = \frac{Ia}{R^2} \tag{1.17}$$

$$\begin{aligned} -\frac{Ia}{R^2} + mg\sin\alpha &= ma \\ -\frac{mR^2a}{2R^2} + mg\sin\alpha &= ma \\ mg\sin\alpha &= \frac{3}{2}ma \\ a &= \frac{2}{3}g\sin\alpha \end{aligned} \tag{1.18}$$

Іродов 1.309



Закон збереження моменту імпульсу:

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} \tag{1.19}$$

Робота сили тертя – зміна енергії:

$$A_{mp} = \frac{I_1\omega_1^2}{2} + \frac{I_2\omega_2^2}{2} - \frac{(I_1 + I_2)\omega^2}{2} =$$

$$= \frac{I_1\omega_1^2}{2} + \frac{I_2\omega_2^2}{2} - \frac{(I_1 + I_2)}{2}\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2 =$$

$$= \frac{I_1\omega_1^2}{2} + \frac{I_2\omega_2^2}{2} - \frac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)} = \tag{1.20}$$

$$= \frac{(I_1\omega_1^2 + I_2\omega_2^2)(I_1 + I_2) - (I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)} =$$

$$= \frac{I_1 I_2(\omega_1^2 + \omega_2^2) - 2I_1 I_2\omega_1\omega_2}{2(I_1 + I_2)} = \frac{I_1 I_2(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$$